CFA L1 自修加強班 數量及統計 講義及題目演練

✓ **PV, FV, PMT, I/Y**

- □ Be sure to get VERY familiar with the use of TI, or it will work on your side throughout the exam
- □ When you define I/Y, you also have to define P/Y
- □ If the question is asking you the implied I/Y, you must set PV the opposite sign against FV
- □ Utilize the BGN mode
- □ You don't have to default the TI every time you come up with the next question. However, you must understand the previous question's setup and do the necessary adjustments in the new question
- □ Know how to use CF/NPV/IRR function
- □ Know how to use natural log (LN) function
- □ Know how to use combination and permutation function

Annuity

1. An investor wants to have $140,000 available at the end of 12 years. This investor plans to make 12 equal year-end payments into an investment that is expected to earn an 8% annual rate of return. The required amount of each year-end payment is closet to:

a. $4,633

b. $6,831

c. $7,377

d. $10,802

2. At an 8% rate of return, how much you have in your investment account on your 65^{th} birthday so you can withdraw $30,000 on that birthday and on each of the next 19 birthdays?

a. $264,540

b. $288,120

c. $294,540

d. $318,108

3. Determine the present value of an annuity of $1,000 paid quarterly for 10 years if the interest rate is 8%

a. $25,653.29

b. $26,840.33
c. $27,355.48
d. $27,977.80

3-1. An individual deposits $10,000 at the beginning of each of the next 10 years, starting today, into an account paying 9% interest compounded annually. The amount of money in the account at the end of 10 years will be closet to:

a. $109,000
b. $143,200
c. $151,900
d. $165,600

Mortgages

4. A bank lends a company $2,000,000 to be repaid in three equal year-end installments. If the bank charges 9% interest, the annual installment payment will be closet to:

a. $666,667
b. $724,871
c. $726,667
d. $790,110

✓ **Uneven Cash Flow Calculation**

5. What is the present value of the following stream of year-end payments discounted at 12% per year?

Year 1	Year 2	Year 3	Year 4
-$100	-$200	-$100	$450

a. -$53.37
b. -$44.65
c. -$33.92
d. -$13.06

6. An investor invests the following amounts at year end

Year 1	$ 1,000
Year 2	$ 4,500
Year 3	$ 2,000
Year 4	$ 4,000

If the invest earns a 10% interest rate, the accumulated amount at the end of Year 4 is closest to

a. $11,500
b. $12,976
c. $13,376
d. $14,274

✓ **Probability Concepts**
Conditional Probability
Unconditional Probability
Joint Probability

The multiplication and addition rules, the total probability rule, and all the expected value and variance/covariance stuff represent the meat of this chapter.

multiplication rule: $P(AB) = P(A|B) \times P(B), P(B) \neq 0$

addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(AB)$

total probability rule: $P(R) = P(R|I) \times P(I) + P(R|I^C) \times P(I^C)$

Remember that I^C is the complement of event I and that the $P(I)+P(I^C)=1$. Hence, the total probability rule simply states that the probability of event R occurring equals the probability that R AND I occur *plus* the probability that R AND I's complement occurs.

18. A dealer in a casino has rolled a five on a single die three times in a row. What is the probability of her rolling another five on the next roll assuming it is a fair die?

a. 0.500
b. 0.200
c. 0.001
d. 0.167

19. Suppose you are modeling GNP, and you believe that the probability that GNP will expand if interest rates fall is 70%; if interest rates stay constant, you believe that there is a 29% chance of expanding GNP; if interest rates increase, you believe that there is a 1% chance of GNP expanding. You think that the likelihood of interest rates failing is 60%; of staying the same is 30%; of increasing is 10%. What is the unconditional probability of GNP expanding?

a. 50.0%

b. 50.8%

c. 50.9%

d. 49.8%

20. An analyst constructs the following probability table for the market and Company's M's stock:

State of the Economy	Probability of the Economic State	Stock Performance	Conditional Probability of Stock Performance
Good	0.4	Good Neutral Poor	0.5 0.25 0.25
Neutral	0.3	Good Neutral Poor	0.5 0.3 0.2
Poor	0.3	Good Neutral Poor	0.4 0.4 0.2

The total probability of good performance for Company M's stock is:

a. 0.10

b. 0.20

c. 0.40

d. 0.47

21. At king investments, 80% of the portfolio mangers are male and 20% female. 95% of the female portfolio mangers have the CFA designation, while 80% of the males have the CFA designation. What is the probability that a portfolio manger would be female and not have a CFA designation?

a. 1%

b. 4%

c. 64%

d. 16%

22. The probability that portfolio A will yield positive annual returns is 0.30. The probability that portfolio B will yield positive annual returns is 0.60. The probability that either portfolio A or B will yield a positive annual return is 0.50. What is the probability that both portfolios realize a positive return?

a. 0.40

b. 0.30
c. 0.20
d. 0.18

Bayes Theorem

if you know the probability that an event B will occur *and* you know the probability that another event A occurs *given* that B has occurred *and* you know the probability that event A occurs, you can compute the probability that B occurs *given* that A has occurred. In other words, you are adjusting your "prior" knowledge of event B with new knowledge about event A.

23. You have developed a set of criteria for evaluating distressed credits. Firms that do not receive a passing score are classed as likely to go bankrupt within 12 months. You gather the following information when validating the criteria:

- Forty percent of the companies to which the test is administered will go bankrupt within 12 months
- Fifty-five percent of the companies to which the test is administered pass it
- The probability that a firm will pass the test (and be classed as a 12-month survivor), given that it will subsequently survive 12 months, is 0.85

Using Bayes' theory, calculate the probability that a firm is a survivor, given that it passes the test?

a. 0.93
b. 0.80
c. 0.20
d. 0.85

✓ Hypothesis Testing

For every statistical test addressed by an LOS, you should be able to compare the computed value of the statistic to the critical value based on the significance level of the test. Be prepared to look up critical values in tables. Remember that AIMR® will *give* you the tables, and they will be abbreviated versions of the tables – not the whole thing. Each question will be self-contained so that you're not flipping back and forth in your test booklet.

Steps	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7	Step 8
Action	Doubt	Set up Hypothesis	Identify Z/t/F/x^2Statistic	Set up Significance level	Set Up Reject Criteria	Calculate Statistic	Compare Statistic with Critical Value	Make Conclusion
Notes	That's why they throw you a question	One tail or Two tails	You may have to memorize the formula	Make sure you use the right one	You must know the degree of freedom and know how to use the D. table			

Note: if you use p-value approach, some steps can be reduced

Difference Between One and Two-tailed Tests

Know the difference between a one and two-tailed test. A one-tailed test is testing whether a value is greater than or less than zero, while a two-tailed test is testing whether a value is different from zero. The difference can be seen in the structure of the hypotheses:

One-tailed test: H_0: $\mu \leqq 0$ versus H_a: $\mu > 0$
One-tailed test: H_0: $\mu = 0$ versus H_a: $\mu \neq 0$

An important point to note is that the decision to use one-tailed versus two-tailed tests *impacts the critical value* used in hypothesis testing. For example, in a one-tailed test at the 5 percent significance level, you are concerned about 5 percent of the observations being in one-tail of the distribution or the other. For a two-tailed test, the 5 percent significance level is effectively divided by two, or in this case 2.5 percent for the upper tail and 2.5 percent for the lower tail. This means that the critical value for a one-tailed test at the 5 percent level of significance is 1.645, while the critical value for a two-tailed test at the 5 percent level of significance is 1.96. *Make sure you are working with the correct critical value when answering questions on the exam.*

Common z-values follow:

Critical Z-values		
Level of significance	*Two-tailed test*	*One-tailed test*
0.10＝10%	±1.65	+1.28 or –1.28
0.055＝5%	±1.96	+1.65 or –1.65

0.01＝1%	±2.58	+2.33 or –2.33

Use the following data to answer Questions 146 to 149

Austin Roberts wants to test if the mean price of houses in the area is equal to or greater than $145,000 A random sample of 36 houses in the area has a mean price of $149,750. The population standard deviation is 24,000. Use the data for the next four questions, assuming a 1% level of significance

35. The alternative hypothesis will be that the population mean is
a. Less than $145,000
b. Not equal to $145,000
c. Greater than or equal to $145,000
d. Greater than $145,000

36. The value of the calculated test statistic is
a. Z=0.67
b. Z=1.19
c. Z=4.00
d. Z=8.13

37. The critical value of the z-statistic is
a. Z=±1.96
b. Z=+2.33
c. Z=-2.33
d. Z= ±2.33

38. At 1% level of significance Robert should ：
a. Accept the null hypothesis
b. Fail to reject the null hypothesis
c. Reject the null hypothesis
d. Neither reject nor fail to reject the null hypothesis

39. A portfolio manager's average return over the last five years is 10%, but a quantitative analyst states that in doing a hypothesis test it cannot be determined with 95% confidence that the return is actually different than zero because the t-statistic is only 1.5. How might the manger show his results are better than zero?
a. test a longer time period (have more observations)
b. Use an F test
c. Invest in stocks with lower standard deviation

d. Use a multiple regression model

40. A pitching machine is calibrated to deliver a fastball at a speed of 98 miles per hour. Every day, a technician samples the speed of 25 fastballs in order to determine if the machine needs adjustment. Today, the sample showed a mean speed of 99 miles per hour with a standard deviation of 1.75 miles per hour. At a 95 percent confidence level, what is the z-value in relation to the critical value? The:

a. critical value exceeds the z-value by 1.3 standard deviation
b. critical value exceeds the z-value by 0.7 standard deviation
c. z-value exceeds the critical value by 0.9 standard deviation
d. z-value exceeds the critical value by 1.5 standard deviation

✓ **Type I and Type II Errors**

When hypothesis testing, there are two possible errors:

- *Type I error:* the rejection of the null hypothesis when it is actually true.
- *Type II error:* the failure to reject the null hypothesis when it is actually false.

The *significance level* is the probability of making a Type I error (rejecting the null when it is true) and is designated by the Greek letter alpha (α). For instance, a significance level of 5 percent (α=0.05) means that there is a 5 percent chance of rejecting a true null hypothesis.

Type I and Type II Errors in Hypothesis Testing		
	True Condition	
Decision	*H_0 is true*	*H_0 is false*
Do not reject H_0	Correct decision	Incorrect decision Type II error
Reject H_0	Incorrect decision Type I error Significance level, α, =P(Type I error)	Correct decision Power of the test =1－P(Type II error)

41. If the significance level of a test is the probability of incorrectly rejecting the null hypothesis, then the "power of a test" is defined as which of the following?

a. the probability of a Type II error
b. the probability of a Type I error

c. the probability of incorrectly accepting the alternative hypothesis
d. the probability of correctly rejecting the null when it is false

42. Suppose we set the criterion for the rejection of the mull that is extremely lax, assuming us that the null will not be rejected. Then, which of the following is/are true?

I. the probability of a type I error is zero
II. the probability of Type II error is zero
III. the significance level of the test is 1

a. II and III only
b. I only
c. I and III only
d. II only

43. What is the probability of making a Type II error if the null hypothesis is actually true?

a. 1-alpha
b. 0
c. 0.05
d. alpha

p-Values

The *p-value* is the probability of obtaining a critical value that is the same as the computed test statistic, assuming the null hypothesis is true. It is the smallest level of significance for which the null hypothesis can be rejected.

There are two decision rules for the *p*-value approach to hypothesis testing:

- Reject H_0 if the *p*-value is less than the significance level of the hypothesis test.
- Do not reject H_0 if the *p*-value is greater than the significance level.

In other words, the smaller (larger) the p-value, the more (less) likely you are to reject the null hypothesis. The same decision will be reached regardless of whether the p-value approach is used or the computed value of the test statistic is compared to the critical value.

44. If the p-value for a statistic is less than the significance level, the decision rule is to:

a. reject the null hypothesis

b. accept the null hypothesis
c. reject the alternative hypothesis
d. conclude that the statistic does not differ significantly from the null hypothesis

Other Tests

The key for *difference of means* testing is that the null hypothesis is referring to two independent samples (i.e., the mean of one population versus the mean of another). Alternatively, the *paired differences* test looks at the behavior of the mean for a particular population (typically over time or related to a fundamental data shift).

My opinion is that the means tests are moderately important. I would memorize the degrees of freedom (*df*) of the tests and be able to state whether to accept or reject a given null hypothesis using a test statistic that has been *given* to you. The key to *all* these statistical test is knowing their *df*. Once you know the *df*, you can look up the critical value of the statistic in the table and compare that value to the computed value. Here are the *df* for each test:

- Difference of means: n_1+n_2-2.
- Paired difference: $n-1$.
- Chi-squared variance test: $n-1$.
- F-test for equality of variance: n_1-1 numerator *df* and n_2-1 denominator *df*.
- Correlation coefficient: n-2
- Regression Coefficient: n-2

✓ **The Regression Equation**

The general form of the linear *regression model* is: $Y_i=b_0+b_1X_i+\varepsilon_i$,

- Y_i and X_i are the i^{th} observation of the dependent and independent variables, respectively.
- b_0 = intercept term (represents the value of Y if X is zero).
- b_1 = slope coefficient (measures the change in Y for a one unit change in X).
- ε_i = residual error of the i^{th} observation.

It is unlikely that you will be asked to calculate a slope coefficient from raw data on the exam, but it is extremely likely that you will be expected to interpret it. The same goes for the intercept term.

Predicted Values

You can count on a predicted value question on the exam, and it should be easy points. You will be given a regression formula in the form of $Y=b_0+b_1X$, and a predicted value for X. From there, simply plug and chug to determine the predicted value for Y.

Use the following data to answer the following four questions:

To help gain a better understanding of the relationship between the return on the common stocks of small companies and the return on the S&P 500 index, you run a simple linear regression to quantify this relationship, using the monthly return on small stocks as the dependent variable and the monthly return on the S&P500 as the independent variable. The results of the regression are shown below:

	Coefficient	Standard error of Coefficient	t-Value
Intercept	1.71	2.95	0.58
S&P 500	1.52	0.13	11.69

The t-statistic critical value at the 0.01 level is 2.58
Standard error of estimate=19.85%
Correlation coefficient=0.7740
N=100
F-Value=101.645 on 1/73 degrees of freedom

47. Use the regression statistics presented above and assume this historical relationship still holds in the future period. If the expected return on the S&P 500 over the next period were 3%, the expected return on small stocks over the next period would be:

a. 4.56%
b. 5.13%
c. 6.27%
d. 6.65%

48. The percent of the variation in the return on the dependent variable (return on small stocks) explained by the return on the independent variable (return on the S&P 500) for the period under study was:

a. 10.07%

b. 19.85%

c. 59.91%

d. 77.40%

49. The regression statistics presented above indicate that at the 0.01 level, the slope coefficient(1.52)

a. and the y-intercept (1.71) are both statistically significant

b. and the y-intercept (1.71) both lack statistical significance

c. is *not* statistically significant, but the y-intercept (1.71) is statistically significant

d. is statistically significant, but the y-intercept (1.71) is *not* statistically significant

50. The regression statistics presented indicate that the standard deviation of the difference between the actual returns on small stocks and the estimate of those returns is:

a. 0.13%

b. 2.66%

c. 2.95%

d. 19.85%

Relationship between R^2, SEE, and the Correlation Coefficient

The *coefficient of determination* (R^2) is formally defined as the percentage of the total variation in the dependent variable explained by the independent variable. It should be noted that in a simple linear regression, the R^2 is simply the square of the correlation coefficient. To give an example of how this relationship works, suppose that portfolio X has a relatively high positive correlation coefficient of 0.9 with the market index. The R^2 value will be $(0.9)^2$ or 0.81. According to the R^2, we can say that 81 percent of the variability in the returns to portfolio X is explained by movements in the index.

$$R^2 = \frac{\text{total variation} - \text{unexplained variation}}{\text{total variation}} = \frac{\text{explained variation}}{\text{total variation}}$$

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

51. Given the following information, determine what percentage of the variation of Y is NOT explained by the regression.

Y = 16 – 4X

$\sigma_y = 12$

$\sigma_x = 15$

$cov_{yx} = 120$

a. 66.7%

b. 33.3%

c. 44.4%

d. 55.6%

Formally, *SEE* is the standard deviation of the actual values for the dependent variable about the regression line. Equivalently, it is the standard deviation of the error terms in the regression. As such, SEE is also referred to as the standard error of the residual, or standard error of the regression, and often

Specified as s_e. SEE will be low if the relationship between the dependent and the independent variable is strong and high if the relationship is weak. The SEE gauges the fit of the regression line. The smaller the standard error, the better the fit of the regression line and the larger the R^2 and r.

$$SEE = \sqrt{s_e^2} = \sqrt{\frac{SSE}{n-2}}$$

Use the following data to answer the next three questions

A regression between the returns on a stock and its industry index gives the following results：

	Coefficient	Standard error	t-value
Intercept	2.1	2.01	1.04
Industry Index	1.9	0.31	6.13

- The t-statistic critical value at the 0.01 level of significant is 2.58
- Standard error of estimate is 15.1
- Correlation coefficient = 0.849

Confidence Intervals and hypothesis test for the slope and dependent value

Another important piece of this chapter is our old friend the confidence interval. You may see a question asking for the *confidence interval for a regression coefficient*. Take the value for the regression coefficient plus or minus the critical *t*-value times the standard error. For a simple linear regression (one independent variable), the *df* of the critical test statistic employed in the confidence interval calculation is n-2. The standard error of the coefficient is the square root of the ratio of the variance of the regression to the variation in the independent variable. Note that as that standard error of the estimate rises, the confidence interval widens. This makes sense since the standard error measures the variability of the data about the regression line, and the more variable the data, the less confidence there is in the regression model to estimate a coefficient.

54-1 An analyst is estimating Microsoft Corp's stock beta using the past 60 trading day's closing prices against S&P 500's. His estimate is 1.3531 and he is wondering if the beta is significantly different from the market beta. The following is the table from his statistics software result:

Regression Statistics			
Multiple R		0.5411	
R-squared		0.2928	
Standard error of estimate		0.0835	
Observations		60	
	Coefficient	Standard Error	*t*-statistics
Alpha	0.00267	0.0117	2.2819
Beta	1.3531	0.2761	

Calculate the t statistic and, using the significant level of 1%, conclude if Microsoft's Beta is different from the market beta?

	t-statistic	Microsoft's Beta
a.	4.0901	different from market beta
b.	2.2819	different from market beta
c.	1.3531	equal to market beta
d.	1.2789	equal to market beta

✓ **ANOVA Tables**

I believe they will give you a simple *ANOVA table* on the exam. ANOVA tables are really cool because you can use them to directly compute:

- R-squared=SSR/SST,
- F=MSR/MSE, and
- The standard error of the estimate (SEE)=square root of (MSE).

Note that:

- SSR measures the sum of squared regression and the regression *df*=*k*=the number of independent variables.
- SSE measures the sum of squared errors and the error *df*=*n*—*k*—*l* (or in a simple linear regression, *n*—2).
- SST equals the total sum of squares and must equal SSE+SSR.
- MSR equals SSR/*k*.
- MSE equals SSE/(*n*—*k*—*l*)

The following three questions tests how to use the ANOVA table:

Simplified ANOVA Table

Source	DF	SS	MSS
Regression	1	185	185.00
Error	16	148	9.25
Total	17	333	194.25

55. Using the ANOVA table, the coefficient of determination is:

a. 0.5556

b. 0.8000

c. 0.9524

d. 0.7454

56. Using the ANOVA table, the standard error of the estimate is:

a. 13.6015

b. 3.0414

c. 12.1655

d. 148.00

57. Using the ANOVA table, the F-statistic is:

a. 0.0500

b. 20.000

c. 4.4721

d. 0.2236

Use the following information for the following four questions:

Source of Variation	Degrees of Freedom	Variation	Variance
Regression	1	20,500	20,500
Error	498	2,500	5.02
Total	499	23,000	

58. What is the standard error of estimate of the regression on which the table is based?

   a. 5.02
   b. 2.24
   c. 1.43
   d. 2.05

59. How many independent variables were used in the regression model?

a. 1
b. 2
c. 499
d. 498

60. Determine the coefficient of determination, the coefficient of correlation, and the standard error of estimate of the regression on which the table is based?

a. 94.4%; 89.1%; 5.02
b. 94.4%; 89.1%; 2.24
c. 89.1%; 94.4%; 5.02
d. 89.1%; 94.4%; 2.24

61. Calculate the F-statistic for the data given in the ANOVA table:

a. 4,084
b. 8.2
c. 498
d. 3,745