Reading 53
Portfolio Concepts

A、Explain mean—variance analysis and its assumptions, and calculate the expected return and the standard deviation of return for a portfolio of two or three assets.

■ Mean-variance portfolio theory (by Harry Markowitz) is based on the idea that the value of investment opportunities can be measured in terms of mean return and variance of return.

■ **Assumptions of mean-variance analysis**

1. All investors are risk averse; they prefer less risk to more for the same level of expected return.
2. Expected returns for all assets are known.
3. The variances and covariances of all asset returns are known.
4. Investors need only know the expected returns, variances, and covariances of returns to determine optimal portfolios. They can ignore skewness , kurtosis, and other attributes of a distribution because returns are often assumed to follow normal distribution.
5. There are no transaction costs or taxes.

■ **Expected return and standard deviation for a two-asset portfolio**

1. The expected return to the portfolio:
   $E(R_P) = w_1E(R_1) + w_2E(R_2)$
2. The standard deviation to the portfolio:

$$\sigma_p = \left(w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2\right)^{1/2}$$

■ **Expected return and standard deviation for a three-asset portfolio**

1. The expected return to the portfolio:
   $E(R_P) = w_1E(R_1) + w_2E(R_2) + w_2E(R_2)$
2. The standard deviation to the portfolio:

$$\sigma_P = \left(w_2^2\sigma_2^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_1 + 2w_2w_2\rho_{2,3}\sigma_1\sigma_3 + 2w_2w_2\rho_{1,2}\sigma_1\sigma_3\right)^{1/2}$$

Example 1

Calculate the expected return and standard deviation of the three-asset portfolio shown in the following figure.

Asset	Apple	HTC	Acer
Amount invested	\$40,000	\$25,000	\$35,000
Expected return	11%	25%	30%
Standard deviation	15%	20%	25%

Correlation

	Apple	HTC	Acer
Apple	-	0.3	0.1
HTC	0.3	-	0.5
Acer	0.1	0.5	-

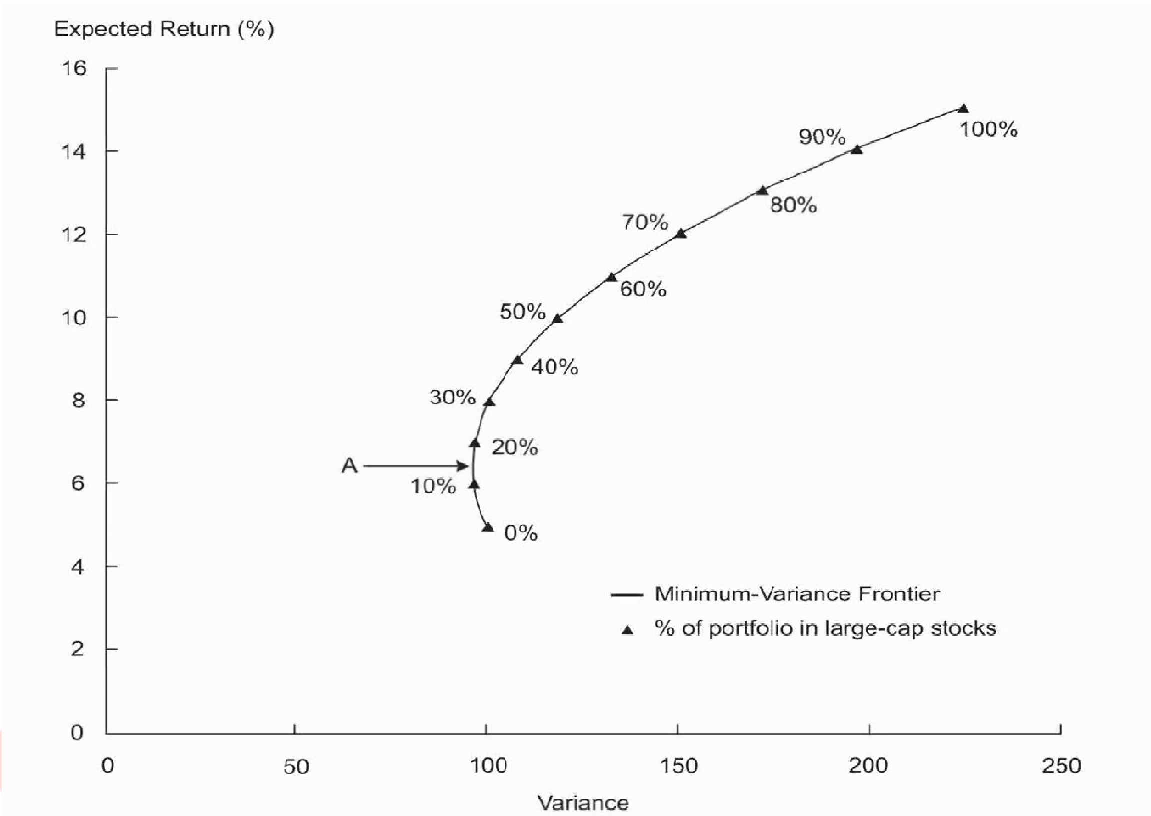
B、Describe the minimum-variance and efficient frontiers, and explain the steps to solve for the minimum-variance frontier.

■ **Minimum-variance frontier**

1. It is the graph of the set of portfolios that have minimum variance for their level of expected return.
2. The portion of the minimum variance frontier beginning with the global minimum-variance portfolio (i.e. point A in the figure 1) and continuing above it is called the Efficient Frontier. It is the positively sloped part of the minimum-variance frontier (i.e. concave portion of the graph). Portfolios lying on the efficient frontier offer maximum expected return for their level of variance of return. Efficient portfolios use risk efficiently.

Figure 1

The possible combinations of risk and return for a portfolio composed of two assets.



■ Steps to solve for the minimum-variance frontier

1. Estimation Step: determine the minimum and maximum expected returns possible with the set of assets.
2. Optimization Step: determine the portfolio weights that minimize the variance of return for a given level of expected return. It is subject to two constraints:
   A. Portfolio expected return equals a pre-specified target return.
   B. Weights must sum to 1.
3. Calculation Step: calculate the expected returns and variances for all the minimum variance portfolios determined in optimization step.

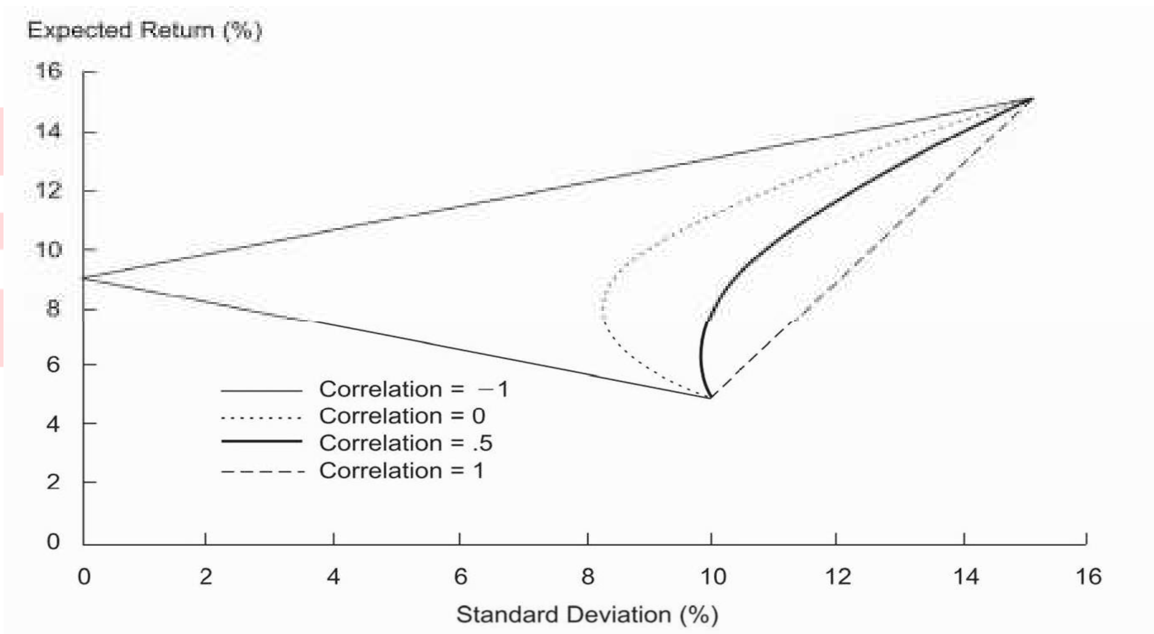
C、Explain the benefits of diversification and how the correlation in a two-asset portfolio and the number of assets in a multi-asset portfolio affect the diversification benefits.

■ **Diversification benefit**

1. It is a reduction in portfolio standard deviation of return through diversification without an accompanying decrease in expected return.

2. When the correlation is +1
   A. The minimum-variance frontier is an upward sloping straight line.
   B. Correlation of +1 means that the return (not just the expected return) on one asset is an exact positive linear function of the return on the other asset. There are no diversification benefits when correlation is +1.
3. Correlation from +1 to -1
   The minimum-variance frontier bows out to the left, in the direction of smaller standard deviation.
4. When the correlation is -1
   Portfolio risk can be reduced to zero, if desired.

Figure 2
The minimum-variance frontier for varied correlations



5. The lower the correlation, the larger the potential benefits to diversification.
6. More assets mean greater diversification benefits. But portfolio risk falls at a decreasing rate as the number of assets included in the portfolio increases.
7. The standard deviation (S.D) of a large well diversified portfolio will get closer and closer to the broad market S.D as the number of assets in the portfolio increases.

D、Calculate the variance of an equally weighted portfolio of n stocks, explain the capital allocation and capital market lines (CAL and CML) and the relation between them, and calculate the value of one of the variables given values of the remaining variables ..

■ Variance of an equally weighted portfolio of n stocks

$$\sigma_p^2 = \frac{1}{n}\bar{\sigma}^2 + \frac{n-1}{n}\overline{Cov} \longrightarrow \sigma_p^2 = \bar{\sigma}^2\left(\frac{1-\rho}{n} + \rho\right)$$

gets closer to the average covariance as n gets large

gets close gets closer to zero as n gets large

The lower the average correlation, the greater the potential benefits of diversification but the greater the number of stocks needed to achieve these diversification benefits.

Example 2

Consider an equally-weighted portfolio comprised of seven assets in which the average asset variance equals 0.31 and the average covariance equals 0.27. What is the variance of the portfolio?

■ **The Capital Allocation Line (CAL)**

1. CAL must be the line from the risk-free rate of return that is tangent to the efficient frontier of risky assets.
2. The CAL is the line of maximum slope that touches the mean-variance frontier.
3. All risk-return lines connecting the risk free rate and portfolios other than tangency portfolio on the efficient frontier will have a flatter slope (lower reward-to-risk ratio) than CAL. Therefore, tangency portfolio is the Optimal Risky Portfolio along the efficient frontier.
4. When combined with risk-free asset, the investor should choose the risky portfolio that maximizes the reward-to-risk ratio.

5. CAL Equation

$$E(R_P) = R_F + \left[\frac{E(R_T) - R_F}{\sigma_{R_T}}\right]\sigma_P$$

Where,

$E(R_P)$ =*expected return on investment (CombinedPortfolio)*

R_F = *risk-free rate of return (It is the intercept of the line)*

$E(R_T)$ = *Expected return on Portfolio T (risky portfolio)*

σ_P = *S.D of Combined Portfolio*

σ_{R_T} = *S.D of Portfolio T (risky portfolio)*

$$\left[\frac{E(R_T) - R_F}{\sigma_{R_T}}\right] = Slpoe\ of\ CAL$$

Example 3

Suppose that the risk-free rate, R_F, is 5 percent; the expected return to an investor's tangency portfolio, $E(R_T)$, is 15 percent; and the standard deviation of the tangency portfolio is 25 percent.

1. How much return does this investor demand in order to take on an extra unit of risk?
2. Suppose the investor wants a portfolio standard deviation of return of 10 percent. What percentage of the assets should be in the tangency portfolio, and what is the expected return?
3. Suppose the investor wants to put 40 percent of the portfolio in the risk -free asset, What is the portfolio expected return? What is the standard deviation?
4. What expected return should the investor demand for a portfolio with a standard deviation of 35 percent?
5. What combination of the tangency portfolio and the risk-free asset does the investor need to hold in order to have a portfolio with an expected return of 19 percent?
6. If the investor has $10 million to invest, how much must she borrow at the risk-free rate to have a portfolio with an expected return of 19%?